Enrollment No: ______________________ Exam Seat No: ______________________

C.U.SHAH UNIVERSITY

Winter Examination-2018

Subject Name : Engineering Mathematics - II

Subject Code : 4TE02EMT3 **Branch: B. Tech (All)**

Semester : 2 **Date : 23/10/2018** **Time : 02:30 To 05:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) The interval of convergence of the logarithmic series

$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - .....\infty$ is

(A) $-1 < x \le 1$ (B) $-1 < x < 2$ (C) $-\infty < x < \infty$ (D) $-1 \le x \le 1$

b) The infinite series $1 + r + r^2 + ... + r^{n-1}$ is divergent if

(A) $|r| < 1$ (B) $|r| > 1$ (C) $r \ge 1$ (D) $r = -1$

c) $\int_0^{\frac{\pi}{2}} \sin^4 x \cos^2 x \, dx$ is equal to

(A) $\frac{1}{16}$ (B) $\frac{1}{32}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{32}$

d) Let $f(b)$ be an odd function in the interval $\left[-\frac{T}{2}, \frac{T}{2}\right]$ with a period *T*, then

$F(x) = \int_a^x f(t) \, dt$ is

(A) periodic (B) non-periodic (C) periodic with period $2T$
(D) periodic with period $4T$

e) If f is continuous, decreasing function on $[1, \infty)$ and $\lim_{x \to \infty} f(x) = 0$, then

$\int_1^\infty f(x) \, dx$ is convergent.

(A) True (B) False

f) $\lceil 4.5 = $ ______

(A) $\frac{\sqrt{\pi}}{16}$ (B) $\frac{105\sqrt{\pi}}{16}$ (C) $\frac{5\sqrt{\pi}}{16}$ (D) none of these

g) Duplication formula: $\lceil n \, \lceil n+\frac{1}{2} =$ ______

(A) $\frac{\sqrt{\pi}\,\lceil n}{2^{2n-1}}$ (B) $\frac{\sqrt{\pi}\,\lceil 2n}{2^{n-1}}$ (C) $\frac{\sqrt{\pi}\,\lceil 2n}{2^{2n-1}}$ (D) $\frac{\sqrt{\pi}\,\lceil n}{2^{n-1}}$

h) If the two tangents at the point are real and distinct the double point is called
(A) a node (B) a cusp (C) a conjugate point (D) none of these

i) $\int_0^1 \int_0^x e^x \, dx \, dy$ is equal to

(A) -1 (B) 1 (C) e (D) e^{-1}

j) The transformations $x+y=u, y=uv$ transform the area element $dy\,dx$ into $|J|\,du\,dv$, where $|J|$ is equal to

(A) 1 (B) u (C) -1 (D) none of these

k) $\int_0^2 \int_1^3 \int_1^2 xy^2 z \, dz\, dy\, dx =$ ______

(A) 62 (B) 26 (C) 24 (D) 42

l) The degree of the differential equation $\frac{d^2y}{dx^2}+3\left(\frac{dy}{dx}\right)^2 = x\log\left(\frac{d^2y}{dx^2}\right)$ is

(A) 1 (B) 2 (C) 3 (D) none of these

m) If $\frac{dy}{dx}+\frac{1}{y\sqrt{1-x^2}}=0$, then which of the following statements is true?

(A) $y+\sin^{-1}x=0$ (B) $y^2+2\sin^{-1}x=c$ (C) $x+\sin^{-1}y=c$ (D) $y=k$

n) The homogeneous differential equation $f_1(x,y)dx+f_2(x,y)dy=0$ can be reduced to a differential equation in which the variables are separated, by the substitution

(A) $y=vx$ (B) $x+y=v$ (C) $xy=v$ (D) $x-y=v$

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) Show that $\sum_{n=1}^{\infty}\frac{1}{n^p}=\frac{1}{1^p}+\frac{1}{2^p}+\frac{1}{3^p}+\ldots\ldots+\frac{1}{n^p}+\ldots\ldots$ is (i) convergent if $p>1$ and (ii) divergent if $p\le 1$. **(5)**

b) Using reduction formula evaluate $\int_0^{\frac{1}{2}} x^3\sqrt{1-4x^2}\, dx$. **(5)**

c) Prove that $\int_0^2 x^4\left(8-x^3\right)^{-\frac{1}{3}} dx = \frac{16}{3}B\left(\frac{5}{3}, \frac{2}{3}\right)$. **(4)**

Q-3 **Attempt all questions** **(14)**

a) Evaluate: $\int_0^1 x^m\left(\log\frac{1}{x}\right)^n dx$ **(5)**

b) Using reduction formula prove that $\int_0^a x^5\left(2a^2-x^2\right)^{-3} dx = \frac{1}{2}\left(\log 2-\frac{1}{2}\right)$. **(5)**

	c)	Discuss the convergence of $\sum \frac{\sqrt{n+1}-\sqrt{n}}{n}$.	**(4)**
Q-4		**Attempt all questions**	**(14)**
	a)	Change the order of integration in $\int_{0}^{a}\int_{a-\sqrt{a^2-y^2}}^{a+\sqrt{a^2-y^2}} dx\,dy$ and evaluate it.	**(5)**
	b)	Test for convergence the series $2+\frac{3}{2}x+\frac{4}{3}x^2+\frac{5}{4}x^3+\ldots(x>0)$ by ratio test.	**(5)**
	c)	Solve: $\frac{dy}{dx}=\cos x\cos y-\sin x\sin y$	**(4)**
Q-5		**Attempt all questions**	**(14)**
	a)	Solve: $\frac{dy}{dx}=\frac{y^3}{e^{2x}+y^2}$	**(5)**
	b)	Evaluate the double integral $\iint_R \left(x^2+y^2\right)\,dxdy$, where R is the square bounded by lines $y=x,\ y=-x,\ x-y=2,\ x+y=2$ using transformations, $u=x+y$ and $v=x-y$.	**(5)**
	c)	Using reduction formula, evaluate $\int_{0}^{\frac{\pi}{6}}\cos^6 3\theta\sin^2 6\theta\,d\theta$.	**(4)**
Q-6		**Attempt all questions**	**(14)**
	a)	Prove that $\int_{-\infty}^{\infty} e^{-k^2x^2}\,dx=\frac{\sqrt{\pi}}{k}$.	**(5)**
	b)	Solve: $\left(x^2-y^2\right)dy=2xy\,dx$	**(5)**
	c)	Evaluate: $\int_{0}^{\frac{\pi}{2}}\int_{0}^{a\sin\theta}\int_{0}^{\frac{a^2-r^2}{a}} r\,dr\,d\theta\,dz$	**(4)**
Q-7		**Attempt all questions**	**(14)**
	a)	Find the asymptotes of the curve $y^3-x^2(6-x)=0$.	**(5)**
	b)	Find the area of the region outside the circle $r=2$ and inside the lemniscate $r^2=8\cos 2\theta$.	**(5)**
	c)	Investigate the convergence of $\int_{2}^{5}\frac{1}{\sqrt{(x-2)}}\,dx$.	**(4)**
Q-8		**Attempt all questions**	**(14)**
	a)	Evaluate: $\int_{2}^{\infty}\frac{x+3}{(x-1)\left(x^2+1\right)}\,dx$	**(5)**
	b)	Trace the curve $r=a(1+\cos\theta)$.	**(5)**
	c)	Find the length of the arc of the Catenary $y=c\ \cosh\left(\frac{x}{c}\right)$ measured from the vertex $(0,\ c)$ to any point on the Catenary.	**(4)**